

$$14) a) A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

$$P(\lambda) = \det \begin{pmatrix} \lambda - 3 & -1 & 0 \\ 0 & \lambda - 3 & -1 \\ 0 & 0 & \lambda - 3 \end{pmatrix} = (\lambda^2 - 6\lambda + 9) \cdot (\lambda - 3)$$

$$= \lambda^3 - 3\lambda^2 - 6\lambda^2 + 18\lambda + 9\lambda - 27$$

$$= \lambda^3 - 9\lambda^2 + 27\lambda - 27$$

Autoval.: $\rightarrow P(\lambda) = 0 \rightarrow \lambda_1 = 3$

$\rightarrow \lambda_2 = 3$

$\rightarrow \lambda_3 = 3$

Para $\lambda = 3$

$$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{cases} y = 0 \\ z = 0 \end{cases} \rightarrow \bar{x} = \alpha \cdot \underbrace{(1, 0, 0)}_{v_1}$$

$$\rightarrow J = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\star (A - 3I)u_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \begin{cases} y=1 \\ z=0 \end{cases} \rightarrow \bar{x} = x \cdot (1, 0, 0) + (0, 1, 0)$$

com $x=0 \rightarrow$ Autovect. $u_2 = (0, 1, 0)$

$$(A - 3I)u_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow \begin{cases} y=0 \\ z=1 \end{cases} \rightarrow \bar{x} = x \cdot (1, 0, 0) + (0, 0, 1)$$

com $x=0 \rightarrow u_3 = (0, 0, 1)$

Soluciones:

$$y(t) = \left\{ e^{3t} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; e^{3t} \cdot \begin{pmatrix} t \\ 1 \\ 0 \end{pmatrix}; e^{3t} \cdot \begin{pmatrix} t^2/2 \\ t \\ 1 \end{pmatrix} \right\}$$

$$\frac{t^2}{2} u_1 + t u_2 + u_3$$